



# Numerical simulation for viscoelastic fibre suspensions

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## *Motivation*

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The orientation of slender particles in suspensions (fibre-reinforced polymeric materials, blood, paint ...) strongly effects the rheological behaviour of the suspensions and the properties of the composite materials.

Flow of fibre suspensions needs to be understood to predict the orientation distribution of the fibres.



## *Description*

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- Simulating the viscoelastic mobility problem for a particle in a non-Newtonian fluid using a Completed Double Layer Boundary Element Method (CDLBEM) (given the driving force and ambient flow, calculate the RBM of the particle and consequently the evolution of microstructure in a complex fluid)
- Dealing with a 3D, time-dependent, viscoelastic flow problem with complex moving boundary



## *Formulation*

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Governing equations for the motion of an incompressible viscoelastic fluid:

$$\nabla \cdot \boldsymbol{\sigma} = \rho \frac{D\mathbf{u}}{Dt}, \quad \nabla \cdot \mathbf{u} = 0, \quad x \in V \quad (1)$$

The total stress tensor in a viscoelastic fluid may be arbitrarily decomposed as

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^N + \boldsymbol{\tau}^v \quad (2)$$



## Formulation

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The balance of momentum becomes

$$\nabla \cdot \boldsymbol{\sigma}^N = \rho \frac{D\mathbf{u}}{Dt} - \nabla \cdot \boldsymbol{\tau}^v \quad (3)$$

The general solution

$$\boldsymbol{\sigma}^N = \boldsymbol{\sigma}^H + \boldsymbol{\sigma}^P \quad (4)$$

where  $\nabla \cdot \boldsymbol{\sigma}^H = 0$

$$\nabla \cdot \boldsymbol{\sigma}^P = \rho \frac{D\mathbf{u}}{Dt} - \nabla \cdot \boldsymbol{\tau}^v$$



## *Homogenous solution*

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The velocity field of the homogenous equation

$$\begin{aligned} U + \boldsymbol{\omega} \times (\mathbf{x} - \mathbf{x}_c) - \mathbf{u}^P(x) &= \mathbf{u}^\infty(x) \\ \varphi(x) + \int_S \mathbf{K}(x, y) \cdot \varphi(y) dS(y), \quad x \in S \end{aligned} \quad (5)$$

Two key ideas of CDLBEM:

- Completion process: capable of representing the general solution.
- Deflation process: guarantee the iterative schemes to be converged



## *Homogenous solution*

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The final equation (in symbolic form)

$$(1 + \mathcal{H})\varphi = \mathbf{b} \quad (6)$$

with the operator

$$H(\cdot) = \int_S \mathbf{K} \cdot (\cdot) dS + \sum_{i,k} \varphi^{(i)} \langle \varphi^{(i)}, \cdot \rangle \quad (7)$$

and  $\mathbf{b}$  is the known vector

When the double layer density  $\varphi$  is known, the velocity at a field point  $\mathbf{x}$  can be calculated



## *Particular solution and Viscoelasticity*

- Particular solution of the inhomogeneous equation:  
Using the radial basic functions

- Upper Convected Maxwell equation for the viscoelastic stress

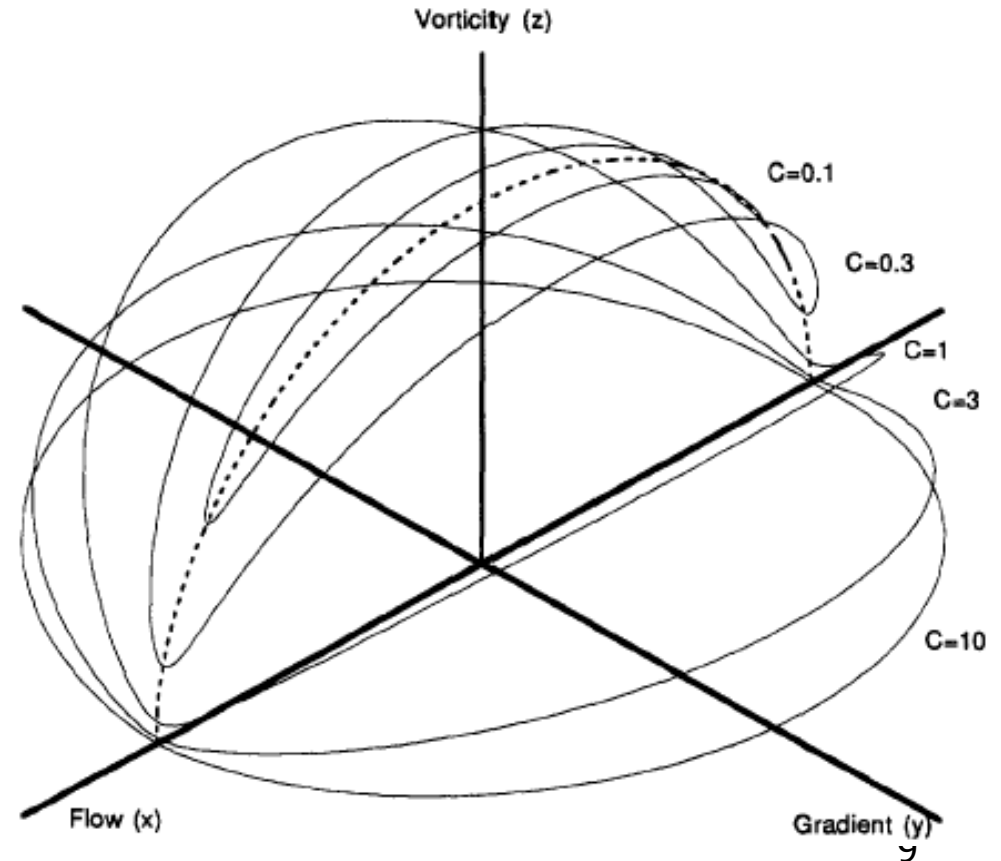
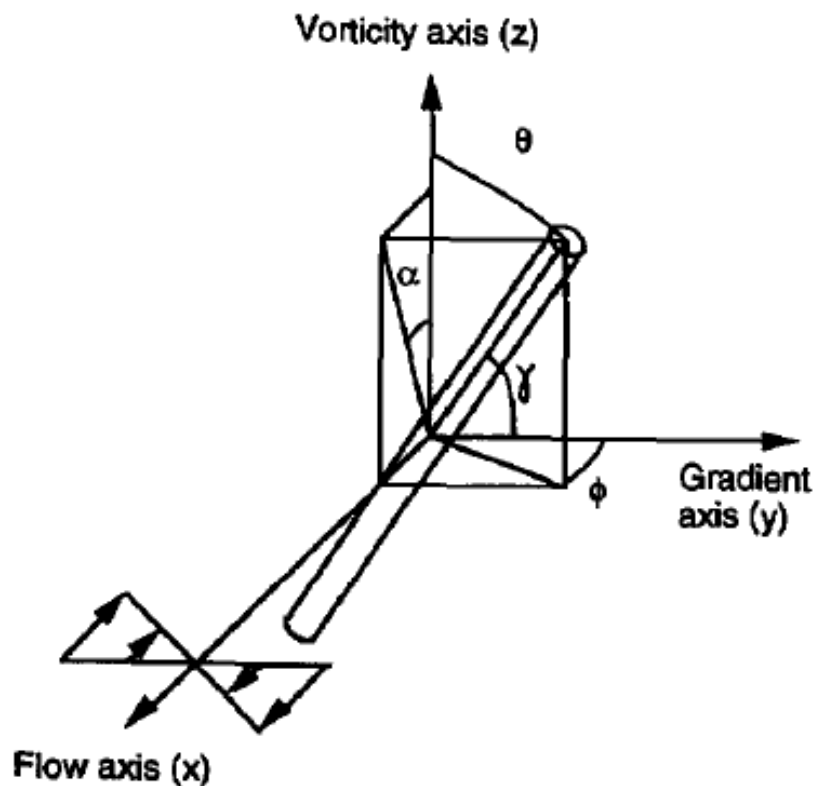
$$\boldsymbol{\tau}^v + \lambda \left( \frac{\partial}{\partial t} \boldsymbol{\tau}^v + \mathbf{u} \cdot \nabla \boldsymbol{\tau}^v - \nabla \mathbf{u}^T \cdot \boldsymbol{\tau}^v - \boldsymbol{\tau}^v \cdot \nabla \mathbf{u} \right) = (\eta_r - 1) \mu \dot{\boldsymbol{\gamma}} \quad (15)$$

which is a reasonable model for viscoelastic fluid at moderate shear rates.



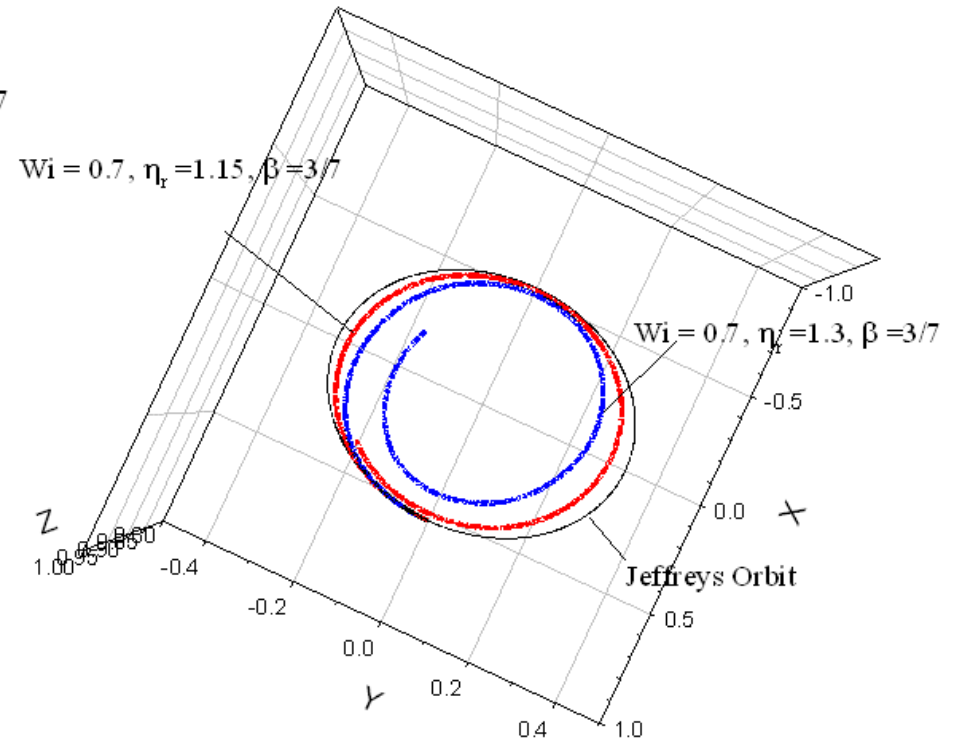
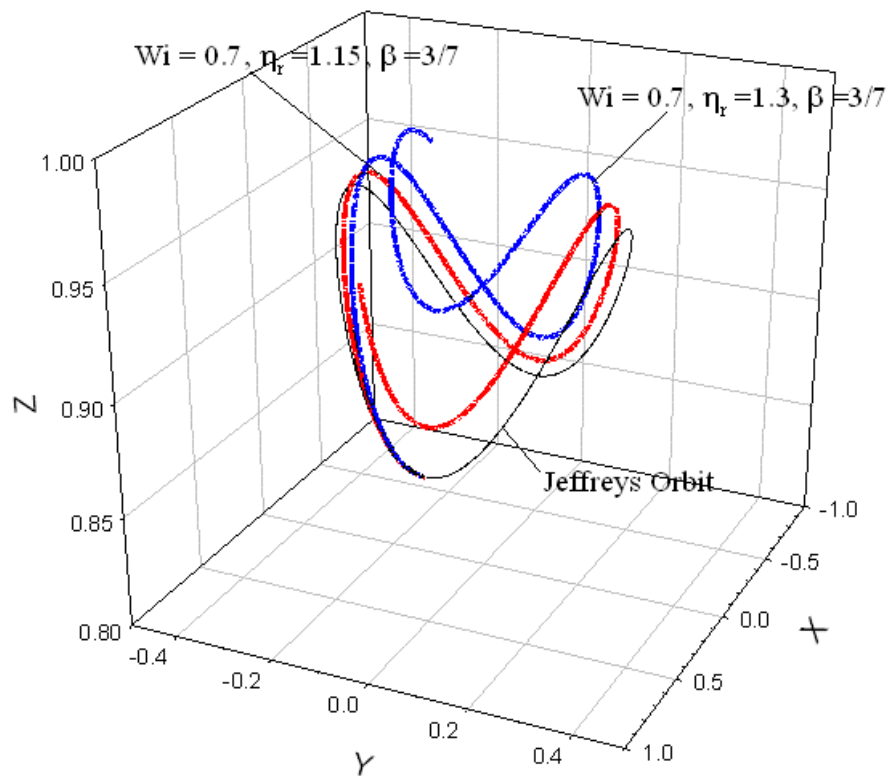
## Results

In a Newtonian shear flow, a slender particle undergoes a periodic motion known as Jeffery's orbit.



# Results

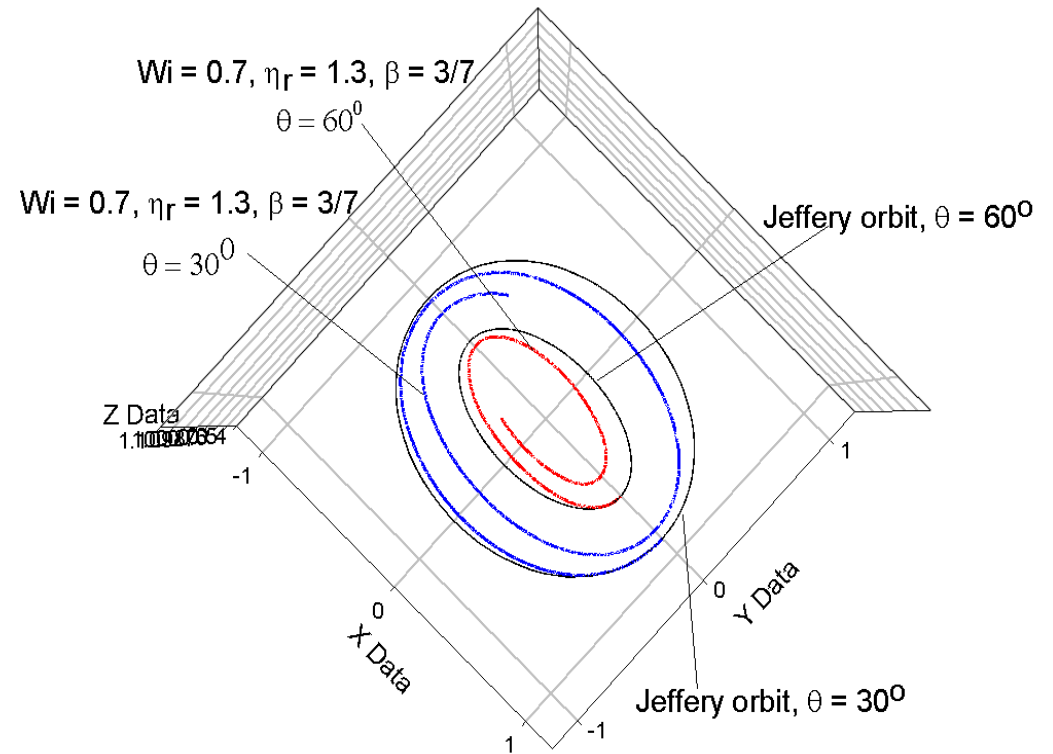
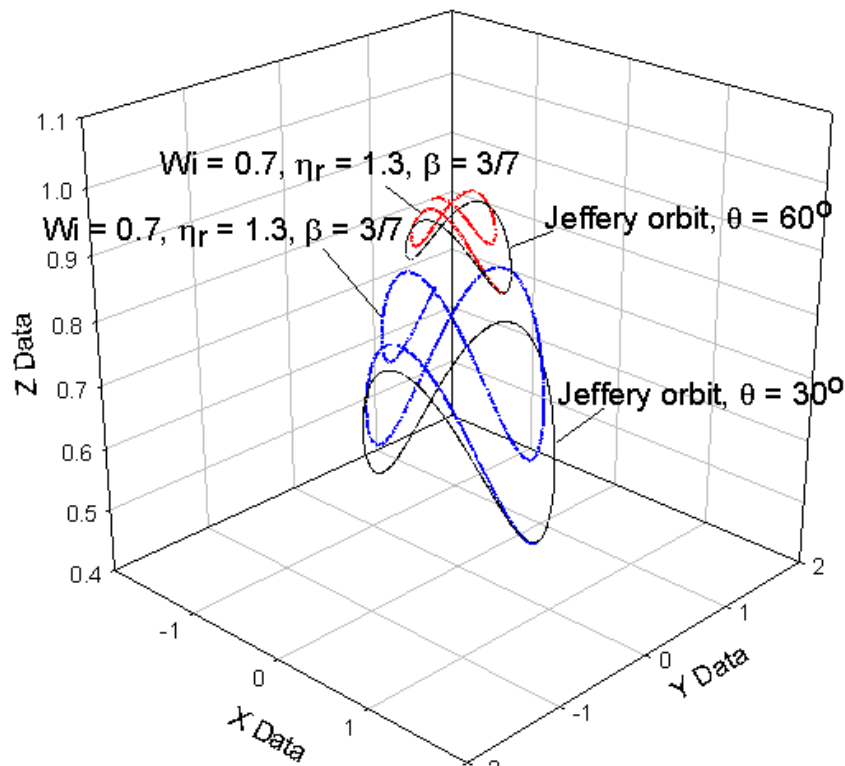
Viscoelastic fluid: the viscoelastic effects causes a drift across Jeffery's orbits towards the vorticity axis.



Numerical orbits with different relative viscosities

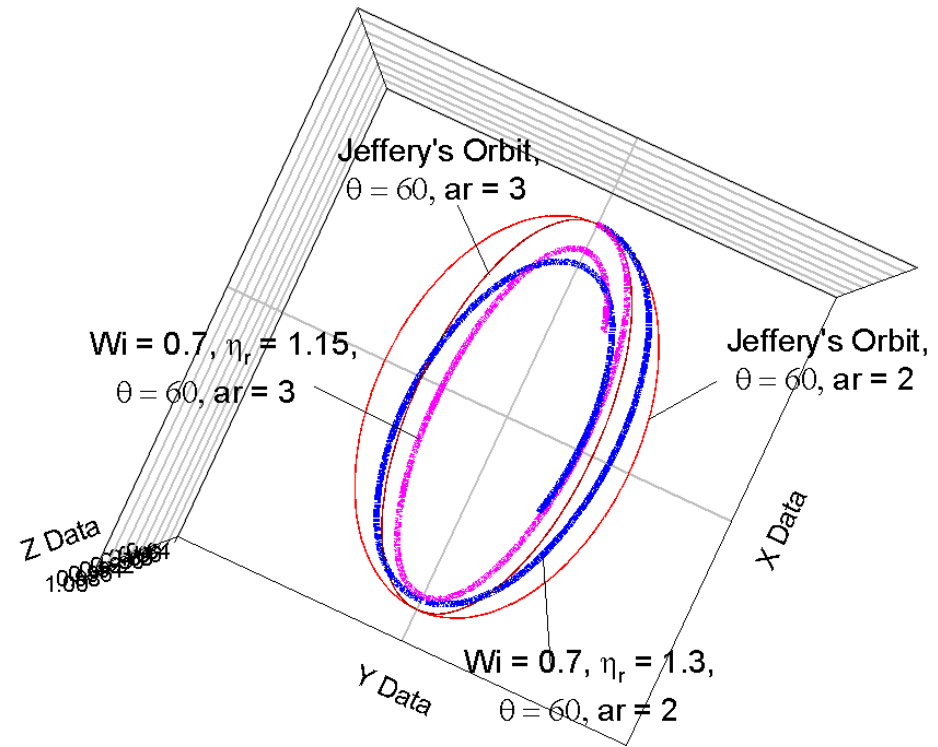
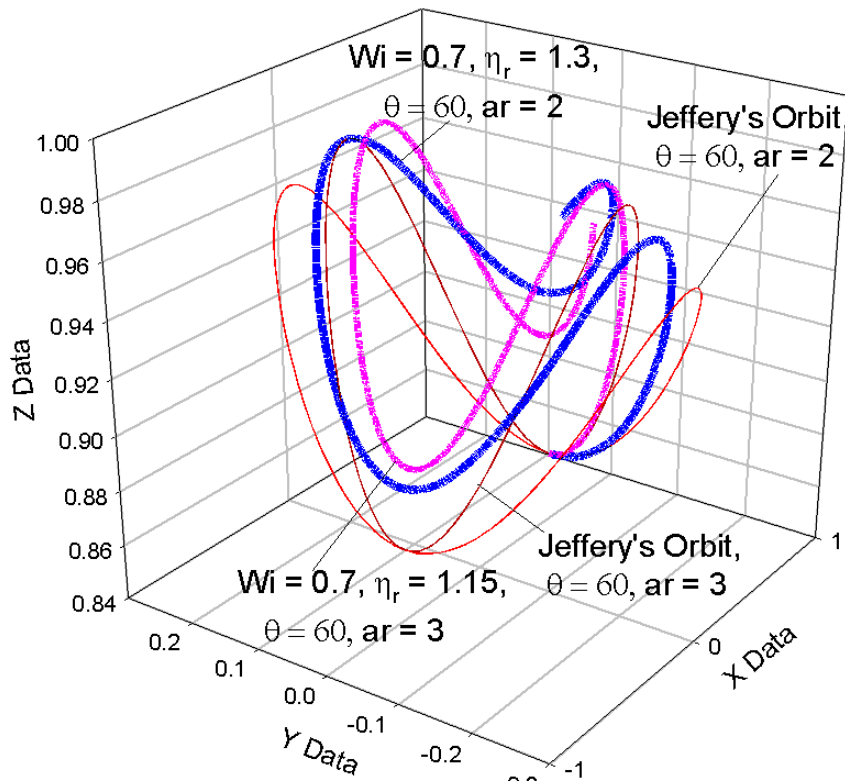
# Results

## Numerical orbits with different initial orientations



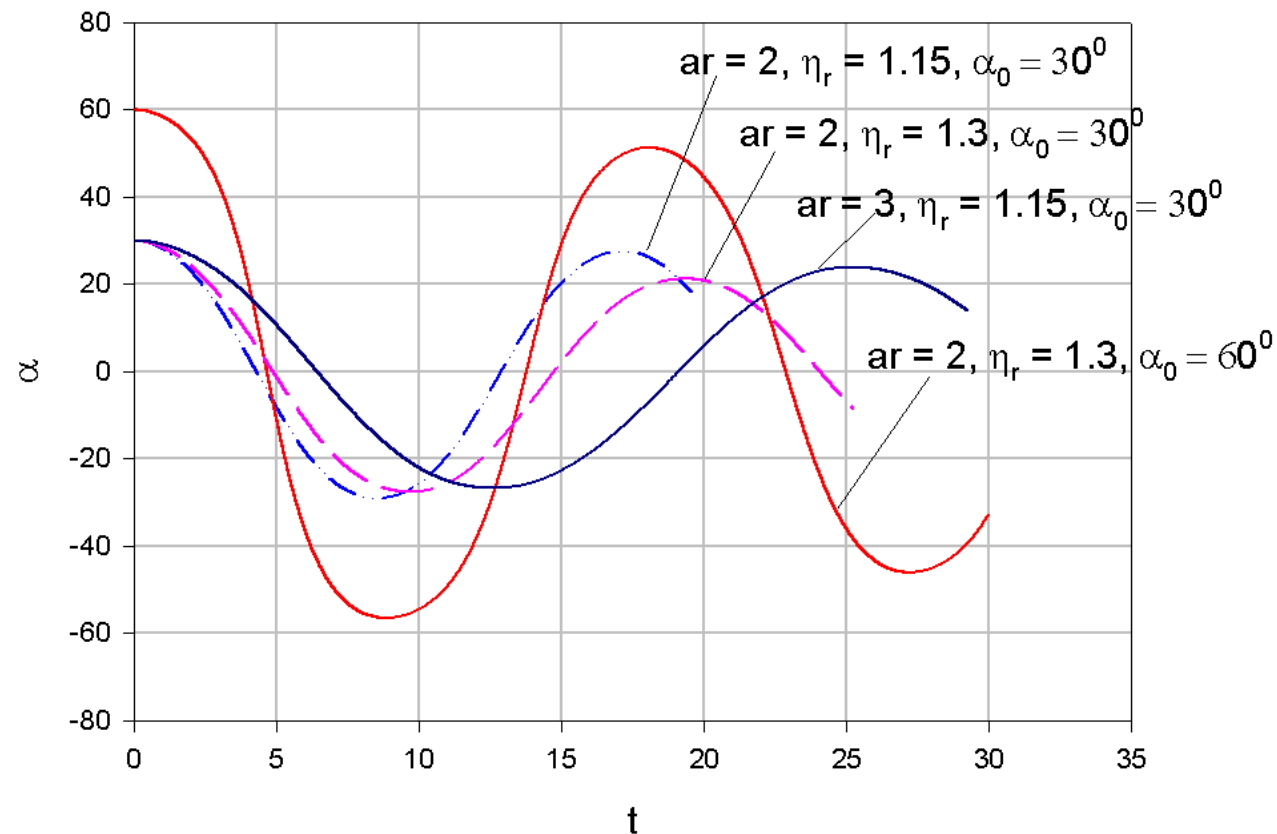
# Results

## Numerical orbits with different aspect ratios



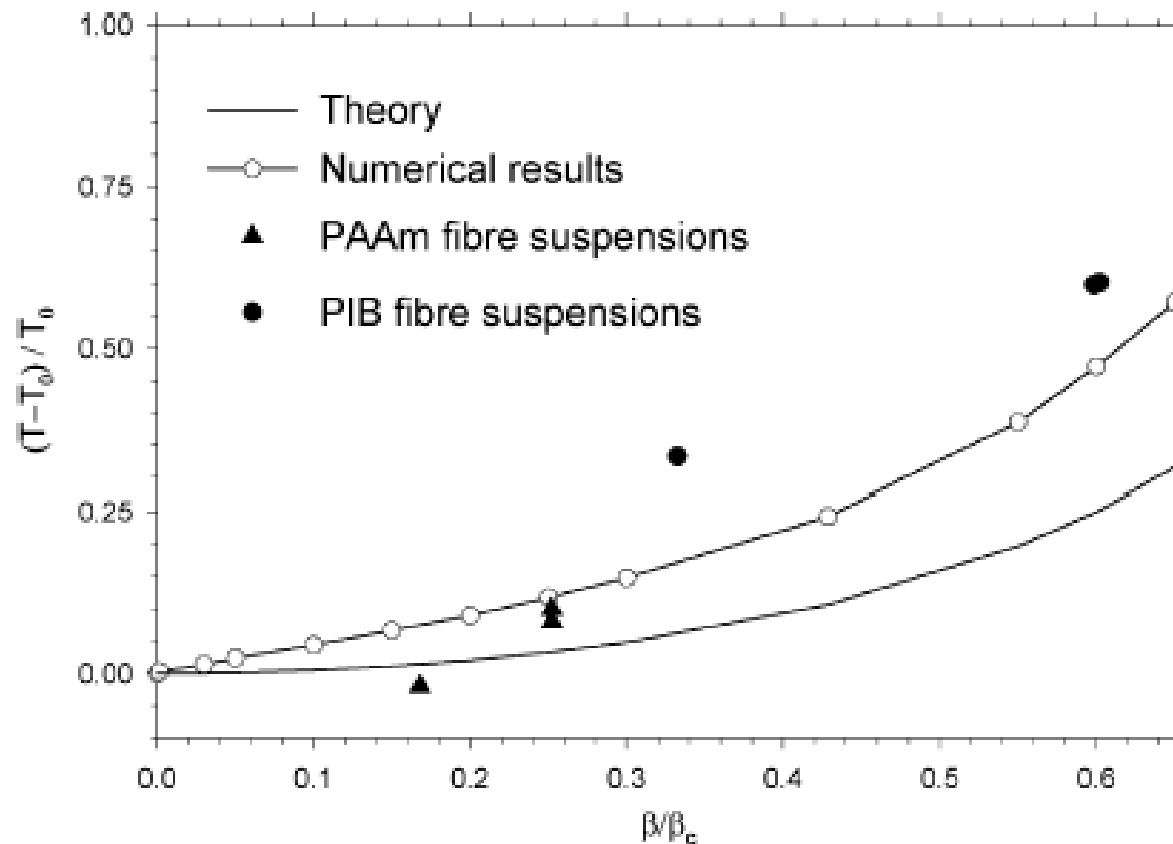
# Results

The angle  $\alpha$  between the projection of the fibre into the velocity-vorticity plane and the vorticity axis decreases with time.



# Results

The viscoelastic stress slows down the rotation of particles in shear flow (period lengthening)



The period of the motion

$$T = \frac{2\pi(a_R + a_R^{-1})}{\gamma(1 - \beta^2 a_R^2 / 4)^{1/2}}$$

Solid line: Harlen and Koch (1993)

Symbols: Iso et al. (1996)



## *Results*

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The two main features of fibre's motion in weakly elastic shear flow:

- The fibre is driven by the viscoelastic force in a spiralling to the vorticity axis;
- The period of the spiral orbit is lengthened by viscoelastic contribution.



## *Summary*

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- The reported CDLBEM is suitable for solving the viscoelastic mobility problem.
- It can be used to flow problems in which the flow has low Re number and weak elasticity.
- The numerical results are used to construct a simple constitutive theory for the fibre suspensions in viscoelastic fluid.